DRE 40XX: Numerical Methods in Finance and Economics

BI Norwegian Business School, Fall 2017

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Preliminary

1 Course Objectives

This is a course in the basic tools of numerical analysis that can be used to address analytically intractable problems in finance and economics. A large class of problems cannot be analyzed with analytical tools, and numerical methods are increasingly expanding the questions we can address.

Numerical methods are vital to all types of applied financial and economic research. The generality with which the techniques will be presented in this course will make them applicable to a wide range of fields, including econometrics, corporate finance, asset pricing, resource economics, labor economics, economic theory, international trade, macroeconomics, finance, game theory, public finance, contract theory and others.

In order to learn how to use computational tools in an informed and intelligent way, this course endeavors to explain not only when and how to use various numerical algorithms but also how and why they work; in other words, the course opens up the "black boxes"and provide the students with a versatile toolbox for their own research.

In the last part of the course we use these tools to analyze numerical problems relevant to the students in the class.

2 Overview

The course have three main parts:

1. Problems on $\mathbb{R}^n$: differentiation, optimization, root-finding, approximation and integration

2. Functional-equations methods: discrete grid methods, finite elements methods, Chebyshev collocation, perturbation methods, linear quadratic methods, Euler equation methods incl. projection methods and weighted residual methods.

3. Applications.

3 Organizing framework for the second part of the course

As an organizing framework for the second part of the course, we will use the definition of competitive, general equilibrium and show how almost all economic models fit into (parts) of this framework and can be solved by a combination and a choice of elementary numerical methods for differentiation, optimization, root-finding, approximation and integration.

Some questions are most satisfactorily answered in partial equilibrium where price functions are given, whereas for other questions it is important to endogenize prices (general equilibrium). We will show the generality of numerical methods by both solving partial equilibrium and general equilibrium problems.

4 Administrative

4.1 Lectures

In order for studens at other institutions to attend the class, we would like to concentrate the classes into as few sessions as possible. At the same time, the learning outcome from this class will be a function of the time spent on problems sets between the sessions. The meetings will therefore be:

1. Thursday October 5$^{\text{th}}$ and Friday October 6$^{\text{th}}$

2. Thursday October 19$^{\text{th}}$ and Friday October 20$^{\text{th}}$

3. Thursday November 2$^{\text{nd}}$ and Friday November 3$^{\text{rd}}$

4. Thursday November 23$^{\text{rd}}$ and Friday November 24$^{\text{th}}$

4.2 Problem sets and final exam

There will be four mandatory problem sets (one per session) and a final term paper (due December 21$^{\text{st}}$).

4.3 Office hours

By appointment.

4.4 Registration

Graduate students at Norwegian universities other than BI Norwegian Business School should contact the PhD administration at BI <phd@bi.no> to register for the course.

5 Recommended preparations

Students are recommended to prepare for the class by completing:

Programming tba

Math preliminaries

- Taylor's theorem, Orders of convergence, Difference equations, Computer arithmetic
  Recommended readings: Chapters 1 and 2 (Math preliminaries and Computer Arithmetic) of Kincaid and Cheney [2001]
- Metric spaces and normed vector spaces, the contraction mapping theorem
  Recommended readings: Chapter 3 of Stokey and Lucas [1989]

6 Software

This course will not teach programming per se, but it will teach and emphasize general principles of programming, such as simplicity, clarity, structure, replicability, and testing. Since one of the objectives of this course is to teach students what is going on inside the "black boxes" of numerical algorithms, students should avoid the use of pre-programmed numerical tool-boxes.

Students may use any programming language to complete the problem sets, including Python, Julia, R, Fortran, C, C++, Matlab, and Gauss. We will recommend Python or Julia, but students are free to choose.[1] Those who intend to solve large-scale problems in the future, it is recommended that you spend the time and effort to learn Fortran or C++.

7 Textbooks

The recommended text for this course is *Quantitative Economics* by Thomas J. Sargent and John Stachurski which is available for free at `http://quant-econ.net/`.

Numerical Recipes: The Art of Scientific Computing. Third Editions is a good suplementary text. Previous editions, *Numerical Recipes in {C, Fortran 77, Fortran 90 }: The Art of Scientific Computing.* are available for free online at: `http://www.nr.com/`. The material covered in these free, previous editions is sufficient for this course.

The descriptions of the algorithms in these books are very instructive also if you choose to use Python, Julia, R or Matlab to solve your problem sets.

Students may also consider buying the following book:

- *Numerical Methods in Economics* by Kenneth L. Judd (MIT Press, 1998).

Other (optional) books students might find useful include:

- *Handbook of Computational Economics (Volume 3)*, edited by Karl Schmedders and Kenneth L. Judd (North-Holland, 2014).
- *Handbook of Computational Economics (Volume 1)*, edited by Hans M. Amman, David A. Kendrick, and John Rust (North-Holland, 1996).
- *Applied Computational Economics and Finance* by Mario J. Miranda and Paul L. Fackler (MIT Press, 2002).
- *Dynamic Economics: Quantitative Methods and Applications* by Jerome Adda and Russell Cooper (MIT Press, 2003).
- *Economic Dynamics: Theory and Computation* by John Stachurski (MIT Press, 2009)
- *Computational Methods for the Study of Dynamic Economies*, edited by Ramon Marimon and Andrew Scott (Oxford University Press, 1999).
- *Numerical Methods in Finance: A MATLAB-Based Introduction* by Paolo Brandimarte (Wiley-Interscience, 2006)

[1] IEEE Spectrum: "The 2017 Top Programming Languages" `http://spectrum.ieee.org/computing/software/the-2017-top-programming-languages`

- *Dynamic General Equilibrium Modelling* by Heer and Maußner (Springer Verlag, 1st edition 2005, 2nd edition 2009)
- *Numerical Methods and Optimization in Finance* by Manfred Gilli, Dietmar Maringer, and Enrico Schumann, (Academic Press 2011).

8 Lecture Plan

1. Thursday October 5th and Friday October 6th

(a) Introduction and overview

(b) Numerical Differentiation
- Numerical Recipes: Chapter 5.7
- Judd: Chapters 1, 2, and 7.7

(c) Root-finding (bisection method, Newton's method, secant method, fixed-point iteration, Gauss-Jacobi, Gauss-Seidel, Brent's method)
- Numerical Recipes: Chapter 9
- Judd: Chapter 5

(d) Minimization in one dimension (Golden Section Search, Newton's method)
- Numerical Recipes: Chapter 10
- Judd: Chapter 4

(e) Minimization in more than one dimension and Quasi-Newton methods.
- Numerical Recipes: Chapters 3 and 6
- Judd: Chapter 6

2. Thursday October 19th and Friday October 20th

(a) Interpolation and approximation of functions, finite elements methods
- Numerical Recipes: Chapters 3 and 6
- Judd: Chapter 6

(b) Interpolation and approximation of functions, (orthogonal) Chebyshev polynomials
- Numerical Recipes: Chapters 3 and 6
- Judd: Chapter 6

(c) Numerical Integration (Newton-Coates quadrature, Gaussian quadrature, Monte Carlo integration)
- Numerical Recipes: Chapter 4 and 7
- Judd: Chapter 7 and 8

(d) Introduction to models of firm dynamics: Hopenhayn economies.
- Classic papers:
  - Hopenhayn [1992]
  - Hopenhayn and Rogerson [1993]

  A good survey on the topic can be found in Hopenhayn [2014]
- Example of recent applications:

- Tom Winberry: "Lumpy Investment, Business Cycles, and Stimulus Policy"
- Pablo Ottonello, Tom Winberry: "How Does Firm Heterogeneity Matter for Aggregate Dynamics? Evidence from Factor Allocation"

3. Thursday November 2nd and Friday November 3rd

(a) Introduction to stationary, incomplete-markets, heterogenous-agents economies: Huggett-Aiyagari Economies

- Aiyagari [1994]
- Ljungqvist and Sargent [2004, Ch. 16-17]
- Huggett [1993], Huggett [1997]

(b) Solving Recursive Individual Decision Problems: Value function iteration, Discretization

- Ljungqvist and Sargent [2004, Ch. 1, 3 and 4.1-4.7]
- Krusell [2006, Ch. 3]
- Burnside [1999]
- Judd [1998, Ch. 12]
- Stokey and Lucas [1989, Ch. 2]
- Hansen and Prescott [1995]

(c) Value function approximation: Finite Element Methods and Chebyshev Collocation

- Ljungqvist and Sargent [2004, Ch. 4.7-4.8]
- Judd [1998, Ch. 6 and 11]
- McGrattan [1996]
- Trick and Zin [1997]

(d) General equilbrium: root finding

(e) Solving Recursive Problems: Value function iteration, Linear Quadratic Methods and Log-Linearization

- Ljungqvist and Sargent [2004, Ch. 5]
- Díaz-Giménez [1999]
- Backus [2004]
- Hansen and Prescott [1995]
- Tallarini [2000]

4. Thursday November 23rd and Friday November 24th

(a) Start: asset-pricing

- Epstein and Zin [1989]
- Bansal and Yaron [2004]
- Backus, Routledge, and Zin [2008]

(b) General perturbation methods

- Judd [1998, Ch. 13-14]

(c) Euler equation methods: Weighted residual method

- Judd [1998, Ch. 11]
- McGrattan [1999]

(d) Dynamics of the distribution function: The Krusell-Smith algorithm, "approximate aggregation"

- Krusell and Smith [1998]
- Reiter (2009): "Solving Heterogeneous-Agent Models by Projection and Perturbation," Journal of Economic Dynamics and Control, 33, 649-665.
- Reiter (2010): "Solving the Incomplete Markets Economy with Aggregate Uncertainty by Backward Induction," Journal of Economic Dynamics and Control, 34, 28-35.
- Young (2010): "Solving the Incomplete Markets Model with Aggregate Uncertainty Using the Krusell-Smith Algorithm and Non-Stochastic Simulations," Journal of Economic Dynamics and Control, 34, 36-41.
- Algan, Allais, Den Haan, and Rendahl (2010) "Solving and Simulating Models with Heterogeneous Agents and Aggregate Uncertainty" *Handbook of Computational Economics*

(e) Other applicatons.

References

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David Backus. Linear-quadratic approximations to dynamic programs. Lecture notes, Stern School of Business, New York University, 2004. `http://pages.stern.nyu.edu/~dbackus/3386/3386notes504.pdf`.

David Backus, Bryan Routledge, and Stan Zin. Recursive preferences. In Steven N. Durlauf and Lawrence E. Blume, editors, *The New Palgrave Dictionary of Economics*. Palgrave, 2008.

Ravi Bansal and Amir Yaron. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59(4), 2004.

Craig Burnside. Discrete state-space methods for the study of dynamic economies. In Ramon Marimon and Andrew Scott, editors, *Computational Methods for the Study of Dynamic Economies*, chapter 5, pages 95–113. Oxford University Press, 1999.

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Gary D. Hansen and Edward C. Prescott. Recursive methods for computing equilibria of business cycle models. In Thomas F. Cooley, editor, *Frontiers of Business Cycle Research*, chapter 2, pages 39–64. Princeton University Press, 1995.

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Hugo A Hopenhayn. Firms, misallocation, and aggregate productivity: A review. *Annual Review of Economics*, 32(6):1–36, 2014.

Mark Huggett. The risk free rate in heterogeneous-agent incomplete insurance economies. *Journal of Economic Dynamics and Control*, 17:953–969, 1993.

Mark Huggett. Idiosyncratic shocks: steady states and dynamics. *Journal of Monetary Economics*, 39(385-403), 1997.

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David R. Kincaid and E. Ward Cheney. *Numerical Analysis. Mathematics of Scientific Computing*. Brooks Cole, 2001.

Per Krusell. Lecture notes for macroeconomics I. Book manuscript, 2006. `http://www.econ.yale.edu/smith/econ510a/book.pdf`.

Per Krusell and Tony Smith. Income and wealth heterogenity in the macroeconomy. *Journal of Political Economy*, 1998.

Lars Ljungqvist and Thomas J. Sargent. *Recursive Macroeconomic Theory*. MIT Press, 2nd edition, 2004.

Ellen R. McGrattan. Solving the stochastic growth model with a finite element method. *Journal of Economic Dynamics and Control.*, 20:19–42, 1996. `http://minneapolisfed.org/research/sr/sr164.pdf`.

Ellen R. McGrattan. Applications of weighted residual mathods to dynamic economic models. In Ramon Marimon and Andrew Scott, editors, *Computational Methods for the Study of Dynamic Economies*, chapter 6, pages 114–142. Oxford University Press, 1999.

William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery. *Numerical Recipes. The Art of Scientific Computing. Third edition*. Cambridge University Press, 2007.

Nancy L. Stokey and Robert E. Lucas. *Recursive Methods in Economic Dynamics*. Harvard University Press, 1989.

Thomas Tallarini. Risk-sensitive real business cycles. *Journal of Monetary Economics*, 45(3), 2000.

Michael Trick and Stanley Zin. Spline approximations to value functions: Linear programming approach. *Macroeconomic Dynamics.*, 1997.